Mathematical literacy

Eva Jablonka
Freie Universität Berlin

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Reviewed Version

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Definition and Development

One of the first written occurrences of the term mathematical literacy was in 1944 in the USA, when a Commission of the National Council of Teachers of Mathematics (NCTM) on Post-War Plans (NCTM (1970/2002), p. 244) required that the school should ensure mathematical literacy for all who can possibly achieve it. Shortly after (in 1950), the term was used again in the Canadian Hope Report (NCTM 1970/2002, p. 401). In more recent times, the NCTM 1989 Standards (NCTM 1989, p. 5) spoke about mathematical literacy and mathematically literate students. Apparently, no definition of the term was offered in any of these texts. The 1989 Standards did, however, put forward five general goals serving the pursuit of mathematical literacy for all students: "(1) That they learn to value mathematics, (2) that they become confident with their ability to do mathematics, (3) that they become mathematical problem solvers, (4) that they learn to communicate mathematically, and (5) that they learn to reason mathematically" (op. cit., p.5). The IEA's Third International Mathematics and Science Study (TIMSS), first conducted in 1995, administered a mathematics and science literacy test to students in their final year of secondary school in 21 countries that aimed "to provide information about how prepared the overall population of school leavers in each country is to apply knowledge in mathematics and science to meet the challenges of life beyond school". The first attempt at an explicit definition appears to be found in the initial OECD framework for PISA (Programme for International Student Assessment) in 1999 (OECD 1999). The definition has been slightly altered a number of times for subsequent PISA cycles. The version for PISA 2012 reads (OECD 2010):

> *Mathematical literacy* is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make well-founded judgments and decisions needed by constructive, engaged and reflective citizens.

In the mathematics education literature, one finds an array of related notions, such as *numeracy*, *quantitative literacy*, *critical mathematical literacy*, *mathemacy*, *matheracy*, as well as *statistical literacy*. While some of these concepts more clearly differ in extension and intension, some authors use "numeracy," "quantitative literacy," and "mathematical literacy" synonymously, whereas others distinguish also between these. While the term "mathematical literacy" seems to be of American descent, the term "numeracy" was coined in the UK. According to Brown et al. (1998, p. 363), it appeared for the first time in the so-called Crowther Report in 1959, meaning scientific literacy in a broad sense, and later obtained wide dissemination through the well-known Cockcroft Report(DES/WO 1982), which stated that its meaning had considerably narrowed by then. There have been further shifts in interpretation since then. A recent, rather wide, definition of “numeracy” can be found in OECD’s PIAAC (Programme for the International Assessment of Adult Competencies) “numeracy” framework: “Numeracy is the knowledge and skills required to effectively manage and respond to the mathematical demands of diverse situations” (PIAAC Numeracy Expert Group 2009, p. 20). The term “quantitative literacy” is yet another term of American descent, going back to the work of Steen (see, e.g., Madison and Steen 2003).

Even though the notions above are interpreted differently by different authors (which suggests a need to pay serious attention to clear terminology), they do have in common that they stress awareness of the usefulness of and the ability to use mathematics in a range of different areas as an important goal of mathematics education. Furthermore, mathematical literacy and related notions are associated with education for the general public rather than with specialized academic training while at the same time stressing the connection between mathematical literacy and democratic participation. As in other combined phrases, such as “statistical literacy” or “computer literacy,” the addition of “literacy” may suggest some level of critical understanding. In South Africa, the pursuit of mathematical literacy has motivated the introduction of a new stand-alone school mathematics subject area available for learners in grades 10–12, which aims at allowing “individuals to make sense of, participate in and contribute to the twenty-first century world – a world characterized by numbers, numerically based arguments and data represented and misrepresented in a number of different ways. Such competencies include the ability to reason, make decisions, solve problems, manage resources, interpret information, schedule events and use and apply technology” (DoBE 2011, p. 8). One motivation for introducing this mathematical subject was to increase student engagement with mathematics.

While “mathematical literacy,” “quantitative literacy,” and “numeracy” focus on mathematics as a tool for solving nonmathematical problems, the notions of *mathematical competence* (and *competencies*) and *mathematical proficiency* focus on what it means to master mathematics at large, including the capacity to solve mathematical as well as nonmathematical problems. The notion of “mathematical proficiency” (Kilpatrick et al. 2001) is meant to capture what successful mathematics learning means for everyone and is defined indirectly through five strands (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition). Furthermore, by referring to individuals’ mental capacities, dispositions, and attitudes, the last two of these strands go beyond mastery of mathematics and include personal characteristics. The notion of “mathematical competence” has been developed, explored, and utilized in the Danish KOM Project (KOM is an abbreviation for “competencies and mathematics learning” in Danish) and elsewhere since the late 1990s (Niss and Højgaar d 2011). Mathematical competence is an individual’s capability and readiness to act appropriately, and in a knowledge-based manner, in situations and contexts in which mathematics actually plays or potentially could play a role. While mathematical competence is the overarching concept, its constituent components are, perhaps, the most important features. There are eight such constituents, called mathematical competencies: mathematical thinking, problem posing and solving, mathematical modeling, mathematical reasoning, handling mathematical representations, dealing with symbolism and formalism, communicating mathematically, and handling mathematical aids and tools. Mathematical competencies do not specifically focus on the learners of mathematics nor on mathematics teaching. Also, no personal characteristics such as capacities, dispositions, and attitudes are implicated in these notions.

Motivations for Introducing Mathematical Literacy

There have always been endeavors amongst mathematics educators to go against the idea that the learning of basic or fundamental mathematics could be characterized solely in terms of facts and rules that have to be known (by rote) and procedures that have to be mastered (by rote). Mathematics educators have found this view reductionist, since it overlooks the importance of understanding when, and under what conditions, it is feasible to activate the knowledge and skills acquired, as well as the importance of flexibility in putting mathematics to use in novel intra- or extra-mathematical contexts and situations. For example, in the first IEA study on mathematics, which later became known as the First International Mathematics Study (FIMS), published in 1967, we read that in addition to testing factual and procedural knowledge and skills related to a set of mathematical topics, it was important to also look into five “cognitive behaviors”: (1) *knowledge and information* (recall of definitions, notations, concepts), (2) *techniques and skills* (solutions), (3) *translation of data into symbols or schema* and vice versa, (4) *comprehension* (capacity to analyze problems and to follow reasoning), and (5) inventiveness (reasoning creatively in mathematics (our italics)). Another example is found in the NCTM document *An Agenda for Action: Recommendations for School Mathematics of the 1980s* (NCTM 1980). The document is partly written in reaction to the so-called “back-to-basics” movement in the USA in the 1970s, which in turn was a reaction to the “new mathematics” movement in the 1950s and 1960s. The document states:

We recognize as valid and genuine the concern expressed by many segments of society that basic skills be part of the education of every child. However, the full scope of what is basic must include those things that are essential to meaningful and productive citizenship, both immediate and future (p. 5).

The document lists six recommendations, including:

2.1. The full scope of what is basic should contain at least the ten basic skill areas […]. These areas are problem solving; applying mathematics in everyday situations; alertness to the reasonableness of results; estimation and approximation; appropriate computational skills; geometry; measurement; reading, interpreting, and constructing tables, charts, and graphs; using mathematics to predict; and computer literacy. (p. 6–7)

2.6 The higher-order mental processes of logical reasoning, information processing, and decision making should be considered basic to the application of mathematics. Mathematics curricula and teachers should set as objectives the development of logical processes, concepts, and language […]. (p. 8)

These examples show that mathematics educators have been concerned with capturing “something more” (in addition to knowledge and skills regarding mathematical concepts, terms, conventions, rules, procedures, methods, theories, and results), which resembles what is indicated by the notion of mathematical literacy as it is, for example, used in the PISA. On the one hand, the arguments for broadening the scope of school mathematics have been utility oriented, based on the observation of students’ lack of ability to use their mathematical knowledge for solving problems that are contextualized in extra-mathematical contexts, in school as well as out of school, an observation corroborated by a huge body of research. On the other hand, the constitution of mathematics as a school discipline in terms of “products” – concepts (definitions and terminology), results (theorems, methods, and algorithms), and techniques (for solving sets of similar tasks) – became challenged. Product-oriented curricula were complemented by, or contrasted with, a conception of mathematics that includes mathematical processes, such as heuristics for mathematical problem solving, mathematical argumentation, constructive and critical mathematical reasoning, and communicating mathematical matters.

There are different views about the amount of mathematical knowledge and basic skills needed for engagement in everyday practices and nonmathematically specialized professions, although it has been stressed that a certain level of proficiency in mathematics is necessary for developing mathematical literacy. The role of general mathematical competencies that transcend school mathematical subareas also has been stressed in the newer versions of conceptualizing mathematical literacy, most prominently in the versions promoted by the OECD-PISA (see above).

Critique and Further Research

Even though the notion of mathematical literacy has gained momentum and is now widely invoked and used in various contexts, it has also encountered different sorts of conceptual and politico-educational criticism.

Some reservations against using the very term “mathematical literacy” concern the fact that it lacks counterparts in several languages. No suitable translation exists, for example, into German and Scandinavian languages, where there are only words for “illiteracy,” which stands for the fundamental inability to read or write *any* text. Indeed, the term “literacy” (both mathematical and quantitative literacy) has been interpreted by some to connote the most basic and elementary aspects of arithmetic and mathematics, in the same way as linguistic literacy is often taken to mean the very ability to read and write, an ability that is seen to transcend the social contexts and associated values, in which reading and writing occurs. However, the demands for reading and writing substantially vary across a spectrum of texts and contexts, as do the social positions of the speakers or readers. The same is true for a range of contexts and situations in which mathematics is used. People’s private, professional, social, occupational, political, and economic lives represent a multitude of different mathematical demands. So, today, for most mathematics educators, the term mathematical literacy signifies a competency far beyond a set of basic skills.

Another critique, going against attempts at capturing mathematical literacy in terms of transferable general competencies or process skills, consists in the observation that such a conception tends to ignore the interests and values involved in posing and solving particular problems by means of mathematics. Jablonka (2003) sees mathematical literacy as a socially and culturally embedded practice and argues that conceptions of mathematical literacy vary with respect to the culture and values of the stakeholders who promote it. Also, de Lange (2003) acknowledges the need to take into account cultural differences in conceptualizing mathematical literacy. There is no general agreement amongst mathematics educators as to the type of contexts with which a mathematically literate citizen will or should engage and to what ends. However, there is agreement that mathematical literate citizens include nonexperts and that mathematical literacy is based on knowledge that is/should be accessible to all.

In the same vein, mathematics educators have empirically and theoretically identified a variety of intentions for pursuing mathematical literacy. For example, Venkat and Graven (2007) investigated pedagogic practice and learners’ experiences in the contexts of South African classrooms, in which the subject mathematical literacy is taught. They identified four different pedagogic agendas (related to different pedagogic challenges) that teachers pursued in teaching the subject. Jablonka (2003), through a review of literature, identifies five agendas on which conceptions of mathematical literacy are based. These are as follows: developing human capital (exemplified by the conception used in the OECD-PISA), maintaining cultural identity, pursuing social change, creating environmental awareness, and evaluating mathematical applications. Some terms have been introduced as alternatives to “mathematical literacy” in order to make the agenda visible. Frankenstein (e.g., 2010) uses *critical mathematical* numeracy, D’Ambrosio (2003) writes about *matheracy*, and Skovsmose (2002) refers to *mathemacy*. Relations of mathematical literacy to scientific and technological literacy have also been discussed (e.g., Keitel et al. 1993; Gellert and Jablonka 2007).

As to the role of mathematical literacy in assessment, discrepancies between actual assessment modes and the intentions of mathematical literacy have been pointed out by researchers in different contexts (Jahnke and Meyerhöfer 2007; North 2010). In the assessment literature, the contexts in which mathematically literate individuals are meant to engage are often referred to in vague or general terms, such as the “real-world,” “everyday life,” “personal life,” “society,” and attempts to categorize contexts often lack a theoretical foundation. Identifying the demands and knowledge bases for mathematically literate behavior in different contexts remains a major research agenda.

As far as the teaching of mathematical literacy is concerned, the transition between unspecialized context-based considerations and problem solutions that employ specialized mathematical knowledge is a continuing concern. Studies of curricula associated with teaching mathematics through and for exploring everyday practices, for example, have usefully drawn on theories of knowledge *recontextualization*.

These observations suggest that the meanings and usages associated with the notion of mathematical literacy and its relatives have not yet reached a stage of universally accepted conceptual clarification nor of general agreement about their place and role. Future theoretical and empirical research and development are needed for that to happen.

References

Brown M, Askew M, Baker D, Denvir H, Millett A (1998) Is the national numeracy strategy research-based? Br J Educ Stud 46:362–385

D’Ambrosio U (2003) The role of mathematics in building a democratic society. In: Madison BL, Steen LA (eds) Quantitative literacy: why numeracy matters for schools and colleges. National Council on Education and the Disciplines, Princeton, pp 235–238

de Lange J (2003) Mathematics for literacy. In: Madison BL, Steen LA (eds) Quantitative literacy: why numeracy matters for schools and colleges. National Council on Education and the Disciplines, Princeton, pp 75–89

Department of Basic Education (2011) Mathematical literacy. National curriculum statement. Further education and training phase. Grades 10–12. http://www.education.gov.za/Curriculum/CurriculumAssessmentPolicyStatements/CAPSFETPhase/tabid/420/Default.aspx. Accessed 8 Nov 2012.

Department of Education and Science/Welsh Office (DES/WO), Committee of Inquiry into the Teaching of Mathematics in Schools (1982) Mathematics counts (“The Cockcroft Report”). HMSO, London

Frankenstein M (2010) Developing critical mathematical numeracy through *real* real-life word problems. In: Gellert U, Jablonka E, Morgan C (eds) Proceedings of the sixth international mathematics education and society conference, Berlin (Mathematics education and society, 6). Freie Universität Berlin, Berlin, pp 248–257

Gellert U, Jablonka E (eds) (2007) Mathematisation and demathematisation: social, philosophical and educational ramifications. Sense Publishers, Rotterdam

Jablonka E (2003) Mathematical literacy. In: Bishop A, Clements MA, Keitel C, Kilpatrick J, Leung FSK (eds) Second international handbook of mathematics education. Kluwer, Dordrecht, pp 75–102

Jahnke T, Meyerhöfer W (eds) (2007) Kritisches zu Pisa & Co, Auflage 2. Franzbecker, Berlin

Keitel C, Kotzmann E, Skovsmose O (1993) Beyond the tunnel vision: analysing the relationship between mathematics, society and technology. In: KC R (ed) Learning from computers: mathematics education and technology. Springer, Berlin, pp 243–279

Kilpatrick J, Swafford J, Findell B (eds) (2001) Adding it up. Helping children learn mathematics. National Academy Press, Washington, DC

Madison BL, Steen LA (eds) (2003) Quantitative literacy. Why numeracy matters for schools and colleges. National Council on Education and the Disciplines, Princeton

NCTM (1970/2002) A history of mathematics education in the United States and Canada, vol 32. National Council of Teachers of Mathematics, Washington, DC

NCTM (1980) An agenda for action. Recommendations for school mathematics of the 1980s. National Council of Teachers of Mathematics, Reston

NCTM (1989) Curriculum and evaluation standards for school mathematics. National Council of Teachers of Mathematics, Reston

Niss M, Højgaard T (eds) (2011) Competencies and mathematical learning. Ideas and inspiration for the development of mathematics teaching and learning in Denmark. Roskilde University, Department of Science, Systems and Models, IMFUFA, Roskilde

North M (2010) How mathematically literate are the matriculants of 2008? A critical review of the 2008 Grade 12 Mathematical Literacy Examinations. In: de Villiers MD (ed) Proceedings of the 16th annual congress of the Association for Mathematics Education of South Africa, vol 1. George Campbell School of Technology, Durban, pp 216–230

OECD (1999) Measuring student knowledge and skills: a new framework for assessment. OECD, Paris

OECD (2010) PISA 2012 mathematics framework. To OECD, November 30, 2010. Draft subject to possible revision after the field trial. www.oecd.org/pisa/pisaproducts/46961598.pdf. Accessed 19 Nov 2012

PIAAC Numeracy Expert Group (2009) PIAAC numeracy: a conceptual framework. OECD Education Working Papers, No 35. OECD, Paris

Venkat H, Graven M (2007) Insights into the implementation of mathematical literacy. In: Setati M, Chitera N, Essien A (eds) Proceedings of the 13th annual national congress of the Association for Mathematics Education of South Africa, vol 1. Uplands College, Mpumulanga, pp 72–83